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Singularity Robust Steering Logic for Redundant Single-Gimbal Control Moment Gyros

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A singularity problem inherent in a system of redundant, single-gimbal control moment gyros (CMGs) is investigated. In particular, a simple yet effective way of passing through, and also escaping from, any internal singularities is presented. The proposed approach is based on the so-called singularity robust inverse, but it utilizes a nondiagonal weighting matrix in the mixed, two-norm and weighted least-squares minimization problem. The proposed steering logic effectively generates deterministic dither signals when the CMG system, even with sensor noises, becomes near singular. Because the proposed CMG steering logic is based on the simple minimum two-norm, pseudoinverse solution, it does not explicitly avoid singularity encounters; rather, it approaches and rapidly transits unavoidable singularities whenever needed. However, the proposed steering logic is not intended for special missions in which prescribed attitude trajectories are to be exactly tracked in the presence of internal singularities.

Introduction

DURING the past three decades, control moment gyros (CMGs), as applied to spacecraft attitude control and momentum management, have been extensively studied. They have also been successfully employed for a variety of space missions. However, there still exist various practical as well as theoretical issues inherent in the use of CMGs. They include system-level tradeoffs (e.g., reaction wheels vs CMGs, single-gimbal vs double-gimbal, etc.), optimal arrangements of CMG arrays (parallel vs skewed or orthogonal mounting, etc.), singularity robust steering logic design (e.g., local vs global methods for singularity avoidance), and computational issues for real-time implementation.

A reaction wheel consists of a spinning rotor, whose spin rate is nominally zero. Its spin axis is fixed to the spacecraft, and its speed is increased or decreased to generate reaction torque about the spin axis. Reaction wheels are conventionally used to control three-axis stabilized spacecraft and smaller satellites. Reaction wheels are the simplest and least expensive of all momentum-exchange actuators; however, most reaction wheels with less than 1 N · m maximum torque have much smaller control torque capability than CMGs with 100–5000 N · m maximum torque.

A CMG contains a spinning rotor with large, constant angular momentum, but whose angular momentum vector direction can be changed with respect to the spacecraft by gimballing the spinning rotor. The spinning rotor is mounted on a gimbal (or a set of gimbals), and torquing the gimbal results in a precessional, gyroscopic reaction torque orthogonal to both the rotor spin and gimbal axes. The CMG is a torque amplification device because small gimbal torque input produces large control torque output on the spacecraft. Because the CMGs are capable of generating large control torques and storing large angular momentum over long periods of time, they are often favored for precision pointing and tracking control of agile spacecraft in low Earth orbit and momentum management of large space vehicles.

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In general, control moment gyros are characterized by their gimballing arrangements and their configurations for the redundancy management and failure accommodation.

There are two basic types of control moment gyros: 1) single-gimbal control moment gyros (SGCMGs) and 2) double-gimbal control moment gyros (DGCMGs). For SGCMGs, the spinning rotor is constrained to rotate on a circle in a plane normal to the gimbal axis. For DGCMGs, the rotor is suspended inside two gimbals, and, consequently, the rotor momentum can be oriented on a sphere along any direction provided that there are no restrictive gimbal stops. The SGCMGs are considerably simpler than DGCMGs from the hardware viewpoint. They offer significant cost, power, weight, and reliability advantages over DGCMGs. However, the gimbal steering problem is much simpler for DGCMGs because of the extra degree of freedom per device.

For the purposes of optimal redundancy management and failure accommodation, many different arrangements of CMGs have been developed in the past. They include four SGCMGs of pyramid configuration, six parallel-mounted SGCMGs, three orthogonally mounted DGCMGs, and four parallel-mounted DGCMGs. The three orthogonally mounted DGCMGs were used in NASA's Skylab, and six parallel-mounted SGCMGs have been successfully installed on the Mir space station. The International Space Station is controlled by four parallel-mounted DGCMGs with two of the four CMGs mounted antiparallel with the other two. However, CMGs have never been used in commercial satellites.

The use of CMGs necessitates the development of CMG steering logic, which generates the CMG gimbal rate commands for the commanded spacecraft control torques. The optimal steering logic is one for which the CMG-generated torques are equal to the commanded spacecraft control torques. One of the principal difficulties in using CMGs for spacecraft attitude control is the geometric singularity problem in which no control torque is generated for the commanded gimbal rates. SGCMG systems are more prone to lock up in singular configurations because of the reduced degrees of freedom. However, reaction wheel systems do not have such a geometric singularity problem inherent to any CMG system.

In this paper the study results of developing singularity robust steering logic for a system of redundant SGCMGs are described. The study objectives were 1) to evaluate a wide variety of existing CMG steering laws in terms of their singularity-avoidance capabilities and 2) to incorporate the existing steering laws with the development of a state-of-the-art control technique for agile spacecraft that are required to maneuver as fast as possible in the presence of the CMG internal singularities, momentum saturation, and gimbal rate limits.

This paper presents an overview of the existing steering laws for a system of redundant SGCMGs (Refs. 1–10). Furthermore, it presents a simple yet effective way of transiting, and also escaping from, any internal singularities. The proposed approach is based on the so-called singularity robust (SR) inverse by Nakamura and Hanafusa[11]; however, it utilizes a nondiagonal weighting matrix in the mixed, two-norm and weighted least-squares minimization problem. The proposed CMG steering logic [U.S. Patents 6,039,290 (Ref. 12) and 6,131,056 (Ref. 13)] effectively generates deterministic dither signals when the CMG system, even with sensor noises, becomes near singular. Because the proposed steering logic is based on the simple minimum two-norm, pseudoinverse solution, it does not explicitly avoid singularity encounters; rather, it approaches and rapidly transits unavoidable singularities whenever needed. The proposed steering logic is not intended for special missions in which prescribed attitude trajectories are to be exactly tracked in the presence of internal singularities. In Ref. 14, the proposed singularity robust steering logic is further integrated with a nonlinear attitude control system of an agile spacecraft for rapid multitarget acquisition and pointing control.

Mathematical Modeling of Rigid Spacecraft with CMGs

In this section the fundamental principles of CMGs are briefly described, as applied to the attitude control of a rigid spacecraft. The objective here is to present a simple mathematical model of a system of redundant CMGs for developing spacecraft attitude control and CMG steering logic.

The rotational equation of motion of a rigid spacecraft equipped with momentum-exchange actuators such as CMGs is simply described by

$$\dot{\boldsymbol{H}} + \boldsymbol{\omega} \times \boldsymbol{H} = \boldsymbol{T}_{\text{ext}} \tag{1}$$

where $\boldsymbol{H} = (H_1, H_2, H_3)$ is the angular momentum vector of the total system expressed in the spacecraft body-fixed control axes; $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$ is the spacecraft angular velocity vector; and $\boldsymbol{T}_{\text{ext}}$ is the external torque vector, including the gravity gradient, solar pressure, and aerodynamic torques, expressed in the body-fixed control axes. The cross product of two vectors is defined in matrix notation as

$$\boldsymbol{\omega} \times \boldsymbol{H} \equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \tag{2}$$

The total angular momentum vector consists of the spacecraft main body angular momentum and the CMG angular momentum, that is, we have

$$\boldsymbol{H} = \boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{h} \tag{3}$$

where $\boldsymbol{J}$ is the inertia matrix of the spacecraft including CMGs and $\boldsymbol{h}$ is the total CMG momentum vector expressed in the spacecraft body-fixed control axes.

Combining Eqs. (1) and (3), we simply obtain

$$(\boldsymbol{J}\dot{\boldsymbol{\omega}} + \dot{\boldsymbol{h}}) + \boldsymbol{\omega} \times (\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{h}) = \boldsymbol{T}_{\text{ext}} \tag{4}$$

Introducing the internal control torque vector generated by CMGs, denoted as $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$, we rewrite Eq. (4) as

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} = \boldsymbol{\tau} + \boldsymbol{T}_{\text{ext}} \tag{5a}$$

$$\dot{\boldsymbol{h}} + \boldsymbol{\omega} \times \boldsymbol{h} = -\boldsymbol{\tau} \tag{5b}$$

In addition to these dynamic equations of motion of a spacecraft equipped with CMGs, we have the following quaternion kinematic differential equations:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{6}$$

The commanded attitude quaternions $(q_{1c}, q_{2c}, q_{3c}, q_{4c})$ and the current attitude quaternions (q_1, q_2, q_3, q_4) are related to the attitude-error quaternions (e_1, e_2, e_3, e_4), as follows:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{7}$$

and $\boldsymbol{e} = (e_1, e_2, e_3)$ is called the quaternion-error vector to be used by an attitude control system.

For a simple spacecraft model, described by Eqs. (5–7), one can design an attitude control and CMG momentum management system. Consequently, the spacecraft control torque input $\boldsymbol{\tau}$ can be assumed to be known for the subsequent CMG steering logic design, and the desired CMG momentum rate is often selected as

$$\dot{\boldsymbol{h}} = -\boldsymbol{\tau} - \boldsymbol{\omega} \times \boldsymbol{h} \tag{8}$$

The CMG angular momentum vector $\boldsymbol{h}$ is in general a function of CMG gimbal angles $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)$, that is, we have

$$\boldsymbol{h} = \boldsymbol{h}(\boldsymbol{\delta}) \tag{9}$$

One approach to the CMG steering logic design is simply to find an inversion of Eq. (9). In this inverse kinematic problem, the task is to determine optimal gimbal angle trajectories that generate the commanded $\boldsymbol{h}$ trajectory while meeting the various hardware constraints, such as the gimbal rate limits and gimbal stops, and while also avoiding singularities.

The second approach involves the differential relationship between gimbal angles and the CMG momentum vector. For such local inversion or tangent methods, the time derivative of $\boldsymbol{h}$ is obtained as

$$\dot{\boldsymbol{h}} = \boldsymbol{A}\dot{\boldsymbol{\delta}} \tag{10}$$

where $\boldsymbol{A} = \boldsymbol{A}(\boldsymbol{\delta})$ is the $3 \times n$ Jacobian matrix defined as

$$\boldsymbol{A} \equiv \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{\delta}} = \left[\frac{\partial h_i}{\partial \delta_j} \right] \tag{11}$$

The CMG steering logic design is then to find an inversion of $\boldsymbol{A}\dot{\boldsymbol{\delta}} = \dot{\boldsymbol{h}}$, that is, to determine the gimbal rates that can generate the commanded $\dot{\boldsymbol{h}}$ while meeting the various hardware constraints, such as the gimbal rate limits and gimbal stops, and also while avoiding singularities. Note that in this formulation of a CMG steering design problem, the gimbal torquer dynamics has been ignored because the gimbal torque input is often much smaller than the control torque output generated by CMGs.

SGCMGs and Pseudoinverse Steering Logic

SGCMG Arrays

Consider a typical pyramid mounting arrangement of four SGCMGs as shown in Fig. 1, in which four CMGs are constrained to gimbal on the faces of a pyramid and the gimbal axes are orthogonal to the pyramid faces. Each face is inclined with a skew angle of β from the horizontal, resulting in gimbal axes with a $(90 - \beta)$ deg inclination from the horizontal. When each CMG has the same angular momentum about its spin-rotor axis and the skew angle is chosen as $\beta = 54.73$ deg, the momentum envelope becomes nearly spherical. This minimally redundant, four CMG configuration with $\beta = 54.73$ deg has been extensively studied in the literature, and it presents a significant challenge for developing singularity robust steering laws.

However, from a momentum storage point of view, the optimal skew angle has been found to be $\beta = 90$ deg, which results in a box configuration of four CMGs. Mounting arrangements of SGCMGs, other than the pyramid mount, are also possible. For example, the six parallel-mounted SGCMGs have been successfully used to control the Mir space station.

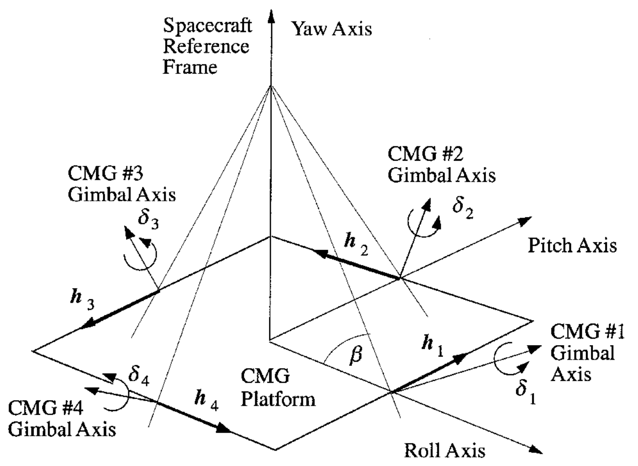

Fig. 1 Pyramid mounting arrangement of four SGCMGs.

Pseudoinverse Steering Logic

For the conventional pyramid mount of four SGCMGs, the total CMG angular momentum vector $\boldsymbol{h}$ is expressed in spacecraft reference frame as

$$\boldsymbol{h} = \sum_{i=1}^{4} \boldsymbol{h}_i(\delta_i)$$

$$= \begin{bmatrix} -c\beta\sin\delta_1 \\ \cos\delta_1 \\ s\beta\sin\delta_1 \end{bmatrix} + \begin{bmatrix} -\cos\delta_2 \\ -c\beta\sin\delta_2 \\ s\beta\sin\delta_2 \end{bmatrix} + \begin{bmatrix} c\beta\sin\delta_3 \\ -\cos\delta_3 \\ s\beta\sin\delta_3 \end{bmatrix} + \begin{bmatrix} \cos\delta_4 \\ c\beta\sin\delta_4 \\ s\beta\sin\delta_4 \end{bmatrix} \tag{12}$$

where $\boldsymbol{h}_i$ is the angular momentum vector of the ith CMG expressed in the spacecraft reference frame, β is the pyramid skew angle, $c\beta \equiv \cos\beta$, $s\beta \equiv \sin\beta$, δ_i are the gimbal angles, and constant unit momentum magnitude for each CMG is assumed without loss of generality.

The time derivative of the CMG angular momentum vector, Eq. (12), can then be obtained as

$$\dot{\boldsymbol{h}} = \sum_{i=1}^{4} \dot{\boldsymbol{h}}_i = \boldsymbol{A}\dot{\boldsymbol{\delta}} \tag{13}$$

where $\boldsymbol{\delta} = (\delta_1, \delta_2, \delta_3, \delta_4)$ is the gimbal angle vector and

$$\boldsymbol{A} = \begin{bmatrix} -c\beta\cos\delta_1 & \sin\delta_2 & c\beta\cos\delta_3 & -\sin\delta_4 \\ -\sin\delta_1 & -c\beta\cos\delta_2 & \sin\delta_3 & c\beta\cos\delta_4 \\ s\beta\cos\delta_1 & s\beta\cos\delta_2 & s\beta\cos\delta_3 & s\beta\cos\delta_4 \end{bmatrix} \tag{14}$$

For a known control torque input $\boldsymbol{\tau}$, the CMG momentum rate command or torque command $\dot{\boldsymbol{h}}$ is chosen as

$$\dot{\boldsymbol{h}} \equiv \boldsymbol{u} = -\boldsymbol{\tau} - \boldsymbol{\omega} \times \boldsymbol{h} \tag{15}$$

and the gimbal rate command $\dot{\boldsymbol{\delta}}$ is then obtained as

$$\dot{\boldsymbol{\delta}} = \boldsymbol{A}^{+}\boldsymbol{u} \tag{16}$$

where $\boldsymbol{A}^{+} = \boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{A}^T)^{-1}$, which is often referred to as the pseudoinverse steering logic. Most CMG steering laws determine the gimbal rate commands with some variant of pseudoinverse.

Singular States

If rank$(\boldsymbol{A}) < 3$ for certain sets of gimbal angles, or equivalently rank$(\boldsymbol{A}\boldsymbol{A}^T) < 3$, the pseudoinverse does not exist, and it is said that the pseudoinverse steering logic encounters singular states. This singular situation occurs when all individual CMG torque output vectors are perpendicular to the commanded torque direction. Equivalently, the singular situation occurs when all individual CMG momentum vectors have extremal projections onto the commanded torque vector direction $\boldsymbol{u}$.

In general, the singularity condition, $\det(\boldsymbol{A}\boldsymbol{A}^T) = 0$, defines a set of surfaces in $\boldsymbol{\delta}$ space, or, equivalently, in $\boldsymbol{h}$ space. The simplest singular state is the momentum saturation singularity characterized by the so-called momentum envelope, which is a three-dimensional surface representing the maximum available angular momentum of CMGs along any given direction. Any singular state for which the total CMG momentum vector is inside the momentum envelope is called internal. For any system of n CMGs, there exist 2^n sets of gimbal angles for which no control torque can be generated along any arbitrary direction. Consequently, for the set of all directions, all of the internal singular states form continuous surfaces both in $\boldsymbol{h}$ space and in $\boldsymbol{\delta}$ space.

There are two types of internal singular states: hyperbolic states and elliptic states. Because the pseudoinverse, $\boldsymbol{A}^{+} = \boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{A}^T)^{-1}$, is the minimum two-norm solution of gimbal rates subject to the constraint $\boldsymbol{A}\dot{\boldsymbol{\delta}} = \boldsymbol{u}$, the pseudoinverse steering logic tends to leave inefficiently positioned CMGs alone, causing the gimbal angles to eventually hang up in singular antiparallel arrangements. That is, it tends to steer the gimbals toward singular states. An approach to avoiding singular states is to introduce null motion into the CMG steering logic. Null motion is a motion of the gimbals that produces no net control torque on the spacecraft. However, the elliptic singular states cannot be escaped through null motion, whereas the hyperbolic singular states can be escaped through null motion.

For the typical pyramid-type system of four CMGs, a set of gimbal angles, $\boldsymbol{\delta} = (90, 0, -90, 0)$ deg, is an elliptic singularity with the singular direction of $\boldsymbol{u} = (\pm 1, 0, 0)$. However, a set of gimbal angles, $\boldsymbol{\delta} = (90, 180, -90, 0)$ deg, is a hyperbolic singularity with the singular direction also along $\boldsymbol{u} = (\pm 1, 0, 0)$.

The impassable elliptic singular states pose a major difficulty with SGCMG systems because they are not escapable without torquing the spacecraft. The impassability is defined locally in $\boldsymbol{\delta}$ space. An impassable surface in $\boldsymbol{h}$ space is not always impassable because it depends on $\boldsymbol{\delta}$. The size of spherical momentum space without impassable singular states is reduced to about one-third of the maximum momentum space of the standard pyramid-type system of four CMGs.

To utilize fully the available momentum space in the presence of impassable singular states, an intelligent steering algorithm needs to be developed that avoids the impassable singular states whenever possible, or rapidly transits unavoidable singularities, while minimizing their effects on the spacecraft attitude control. In the next section, such an intelligent, yet simple and effective, steering algorithm will be described.

SR Steering Logic

Conventional Singularity-Avoidance Steering Logic

Equation (16) can be considered as a particular solution to Eq. (13). The corresponding homogeneous solution is then obtained through null motion such that $\boldsymbol{A}\boldsymbol{n} = 0$, where $\boldsymbol{n}$ denotes the null vector spanning the null space of $\boldsymbol{A} \in R^{3\times 4}$. Without loss of generality, we consider here the minimally redundant, four CMG system shown in Fig. 1.

The general solution to Eq. (13) is then given by

$$\dot{\boldsymbol{\delta}} = \boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{A}^T)^{-1}\boldsymbol{u} + \rho\boldsymbol{n} \tag{17}$$

where ρ represents the amount of null motion to be properly added.

The amount of null motion may be chosen as[1]

$$\rho = \begin{cases} m^6 & \text{for} \quad m \geq 1 \\ m^{-6} & \text{for} \quad m < 1 \end{cases} \tag{18}$$

where $m = \sqrt{[\det(\boldsymbol{A}\boldsymbol{A}^T)]}$ is the singularity measure, $\boldsymbol{n} = (C_1, C_2, C_3, C_4)$ is the Jacobian null vector, $C_i = (-1)^{i+1}M_i$ is the order three Jacobian cofactor, $M_i = \det(\boldsymbol{A}_i)$ is the order three Jacobian minor, and $\boldsymbol{A}_i = \boldsymbol{A}$ with the ith column removed. This choice of scaling factor ρ arises from the representation of m as a measure of distance from singularity, as well as from

$$\det(\boldsymbol{A}\boldsymbol{A}^T) = \sum_{i=1}^{4} M_i^2 = \boldsymbol{n}^T\boldsymbol{n} \tag{19}$$

This nondirectional null-motion approach introduces substantial null motion even when the system is far from being singular and tries to prevent the gimbal angles from settling into locally optimal configurations, which may eventually result in a singularity. However, this approach does not guarantee singularity avoidance and has few potential shortcomings, as discussed in Ref. 1.

Although the null vector can be obtained through a variety of ways (e.g., using singular value decomposition, a projection operator, or the generalized cross or wedge product), it is often determined as

$$n = [I - A^{+}A]d \tag{20}$$

where $A^{+} = A^{T}(AA^{T})^{-1}$, I is an identity matrix, $[I - A^{+}A]$ is a projection matrix, and d is an arbitrary nonzero vector. (A symmetric matrix B is called a projection matrix if $B^{2} = B$.)

A variety of analytic and heuristic approaches have been developed in the past to determine a proper null motion for singularity avoidance, that is, to properly select ρ and d. In a gradient-based method, a scalar function $f(\delta)$ is defined such that its gradient vector points toward the singular directions. The vector d is then chosen as

$$d = \left(\frac{\partial f}{\partial \delta_1}, \ldots, \frac{\partial f}{\partial \delta_4}\right) \tag{21}$$

and the scalar function is often selected as the inverse of the square of the singularity measure, that is,

$$f(\delta) = 1/\det(AA^{T}) \tag{22}$$

and the scalar ρ is determined by minimizing $f(\delta)$ in the null vector direction.

The gradient-based method does not always work in directly avoiding internal singularities. Consequently, an indirect singularity-avoidance steering law of feedback control form, which adds null motion to steer toward a set of desired gimbal angles, can also be employed as

$$\dot{\delta} = A^{+}u + \rho[I - A^{+}A](\delta^{*} - \delta) \tag{23}$$

where δ^{*} denotes a set of desired gimbal angles and ρ is a positive scalar. However, this approach does not guarantee singularity avoidance.

SR Inverse

Given $A\dot{\delta} = u$, the pseudoinverse solution, also called the Moore–Penrose inverse solution, is given by

$$\dot{\delta} = A^{+}u \tag{24}$$

where $A^{+} = A^{T}(AA^{T})^{-1}$, which is the minimum two-norm solution of the following constrained minimization problem

$$\min_{\dot{\delta}} \|\dot{\delta}\|^{2} \qquad \text{subject to} \qquad A\dot{\delta} = u \tag{25}$$

where $\|\dot{\delta}\|^{2} = \dot{\delta}^{T}\dot{\delta}$.

The pseudoinverse is a special case of the weighted minimum two-norm solution

$$\dot{\delta} = A^{+}u \tag{26}$$

where $A^{+} = Q^{-1}A^{T}[AQ^{-1}A^{T}]^{-1}$, of the following constrained minimization problem:

$$\min_{\dot{\delta}} \|\dot{\delta}\|_{Q}^{2} \qquad \text{subject to} \qquad A\dot{\delta} = u \tag{27}$$

where $\|\dot{\delta}\|^{2} = \dot{\delta}^{T}Q\dot{\delta}$ and $Q = Q^{T} > 0$.

If rank$(A) < 3$ (i.e., if A is not of full rank), then $\det(AA^{T}) = 0$ and the pseudoinverse A^{+} does not exist. To determine an inverse solution of $A\dot{\delta} = u$ even when the rank of A is less than 3, consider the following mixed, two-norm and least-squares minimization problem[11,15]:

$$\min_{\dot{\delta}}\{(A\dot{\delta} - u)^{T}P(A\dot{\delta} - u) + \dot{\delta}^{T}Q\dot{\delta}\} \tag{28}$$

where P and Q are positive definite weighting matrices, that is, $P = P^{T} > 0$ and $Q = Q^{T} > 0$. The so-called singularity robust inverse solution can be obtained as

$$\dot{\delta} = A^{\#}u \tag{29}$$

where $A^{\#} = [A^{T}PA + Q]^{-1}A^{T}P$, where $[A^{T}PA + Q]$ is positive definite and, thus, nonsingular for any set of gimbal angles.

If $Q = 0$, the singularity robust inverse solution has the form of the classical, weighted least-squares solution

$$A^{\#} = [A^{T}PA]^{-1}A^{T}P \tag{30}$$

which exists only for $A \in R^{m\times n}$ $(m \geq n)$ and rank$(A) = n$.

Note that in the original development of the SR inverse by Nakamura and Hanafusa,[11] the weighting matrices P and Q are assumed as diagonal matrices. In the literature, it is further assumed that $P = I_3$ and $Q = \lambda I_4$ resulting in a singularity robust inverse solution of the form

$$\dot{\delta} = A^{\#}u \tag{31}$$

where

$$A^{\#} = \left[A^{T}A + \lambda I_4\right]^{-1}A^{T} \equiv A^{T}\left[AA^{T} + \lambda I_3\right]^{-1} \tag{32}$$

and λ is a scalar to be properly selected. For example, Nakamura and Hanafusa[11] suggested that the scalar λ be adjusted as a function of the singularity measure, $m = \sqrt{[\det(AA^{T})]}$, as follows:

$$\lambda = \begin{cases} 0 & \text{for} \quad m \geq m_0 \\ \lambda_0(1 - m/m_0)^2 & \text{for} \quad m < m_0 \end{cases} \tag{33}$$

where λ_0 is a constant scalar at singular points and m_0 a singularity threshold that defines the boundary of the neighborhood of singular points. A different choice of the scale factor λ is also possible as

$$\lambda = \lambda_0 \exp[-\mu \det(AA^{T})] \tag{34}$$

where μ is a scalar to be properly selected.

Most existing pseudoinverse-based, local inversion, or tangent methods, including the SR inverse of the form of Eq. (32), do not always guarantee singularity avoidance and often unnecessarily constrain the operational momentum envelope of SGCMG systems. A SGCMG system, even with the SR inverse of the form of Eq. (32) and sensor noises, can become singular. Furthermore, if it does become singular and a control torque is commanded along the singular direction even in the presence of sensor noises, the system becomes trapped in the singular state because the SR inverse of the form of Eq. (32) is unable to command nonzero gimbal rates. This property of the SR inverse will be further discussed using the singular value decomposition later in this section. To guarantee successful singularity avoidance throughout the operational envelope of SGCMGs, a global method was also suggested by Paradiso.[5] However, any global method requires extensive computations that may not be practical for real-time implementation.

Generalized SR Inverse

We now present a simple yet effective way of passing through, and also escaping from, any internal singularities. The proposed CMG steering logic [U.S. Patents 6,039,290 (Ref. 12) and 6,131,056 (Ref. 13)] is mainly intended for typical reorientation maneuvers in which precision pointing or tracking is not required during reorientation maneuvers, and it fully utilizes the available CMG momentum space in the presence of any singularities. Although there are special missions in which prescribed attitude trajectories are to be exactly tracked in the presence of internal singularities, most practical cases will require a tradeoff between robust singularity transit/escape and the resulting, transient pointing errors.

A generalized singularity robust inverse developed in Refs. 12 and 13 can be represented as

$$\begin{aligned} A^{\#} &= \left[A^T P A + \lambda I_4\right]^{-1} A^T P \\ &= A^T\left[P A A^T + \lambda I_3\right]^{-1} P \\ &= A^T[A A^T + \lambda P^{-1}]^{-1} \\ &= A^T[A A^T + \lambda E]^{-1} \end{aligned} \tag{35}$$

where

$$P^{-1} \equiv E = \begin{bmatrix} 1 & \epsilon_3 & \epsilon_2 \\ \epsilon_3 & 1 & \epsilon_1 \\ \epsilon_2 & \epsilon_1 & 1 \end{bmatrix} > 0 \tag{36}$$

The scalar λ and the off-diagonal elements ϵ_i are to be properly selected such that $A^{\#}u \neq 0$ for any nonzero constant u.

Note that there always exists a null vector of $A^{\#}$ because $\text{rank}(A^{\#}) < 3$ for any λ and ϵ_i when the Jacobian matrix A is singular. Consequently, a simple way of guaranteeing that $A^{\#}u \neq 0$ for any nonzero constant u command is to modulate ϵ_i continuously, for example, as follows:

$$\epsilon_i = \epsilon_0 \sin(\omega t + \phi_i) \tag{37}$$

where the amplitude ϵ_0, the modulation frequency ω, and the phases ϕ_i need to be appropriately selected.[12,13]

It is emphasized that the generalized SR inverse (35) is based on the mixed, two-norm and weighted least-squares minimization although the resulting effect is somewhat similar to that of artificially misaligning the commanded control torque vector from the singular vector directions. Because the proposed steering logic is based on the minimum two-norm, pseudoinverse solution, it does not explicitly avoid singularity encounters; rather, it approaches and rapidly transits unavoidable singularities whenever needed. The proposed logic effectively generates deterministic dither signals when the system becomes near singular. Any internal singularities can be escaped for any nonzero constant torque commands using the proposed steering logic.

Next, the singular value decomposition technique will be applied to the various pseudoinverse solutions, including the SR inverse. The Moore–Penrose generalized pseudoinverse will also be described for a possible application to the CMG steering problem.

Singular Value Decomposition of A, A^+, and $A^{\#}$

Consider a 3×4 Jacobian matrix A of rank 3 without loss of generality. For such $A \in R^{3\times 4}$ of rank 3, there exist orthonormal matrices $U \in R^{3\times 3}$ and $V \in R^{4\times 4}$ such that $U^T U = I_3$ and $V^T V = I_4$, and

$$A = U \Sigma V^T \tag{38}$$

where

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 \end{bmatrix} \tag{39}$$

The positive numbers σ_1, σ_2, and σ_3 are called the singular values of A.

From Eq. (38), we have

$$(A A^T) U = U(\Sigma \Sigma^T) \quad \text{or} \quad (A A^T) u_i = \sigma_i^2 u_i \tag{40a}$$

$$(A^T A) V = V(\Sigma^T \Sigma) \quad \text{or} \quad (A^T A) v_i = \sigma_i^2 v_i \tag{40b}$$

where

$$U = [u_1 \quad u_2 \quad u_3] \tag{41a}$$

$$V = [v_1 \quad v_2 \quad v_3 \quad v_4] \tag{41b}$$

and U is the orthonormal modal matrix of $A A^T$, whereas V is the orthonormal modal matrix of $A^T A$. The modal form of $A A^T$ is $\Sigma\Sigma^T$, whereas the modal form of $A^T A$ is $\Sigma^T \Sigma$. The columns of U are called the left singular vectors of A or the orthonormal eigenvectors of $A A^T$. Similarly, the columns of V are called the right singular vectors of A or the orthonormal eigenvectors of $A^T A$.

The singular values of A are, thus, defined to be the positive square roots of the eigenvalues of $A A^T$, that is,

$$\sigma_i(A) = \sqrt{\lambda_i(A A^T)} \tag{42}$$

where $\lambda_i(A A^T)$ denotes the ith eigenvalue of $A A^T$ and all $\lambda_i(A A^T) \geq 0$. Furthermore, we have

$$\det(A A^T) = (\sigma_1 \sigma_2 \sigma_3)^2 \tag{43}$$

The largest and smallest singular values of A, denoted by $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$, respectively, are given by

$$\bar{\sigma}(A) = \sqrt{\lambda_{\max}(A A^T)} \tag{44a}$$

$$\underline{\sigma}(A) = \sqrt{\lambda_{\min}(A A^T)} \tag{44b}$$

The choice of $A A^T$ rather than $A^T A$ in the definition of singular values is arbitrary. Only the nonzero singular values are usually of real interest, and their number is the rank of the matrix. The matrix $A^T A$ is a square matrix of order four and is a positive semidefinite symmetric matrix.

From Eq. (38), A can be expanded in terms of the singular vectors u_i and v_i as follows:

$$\begin{aligned} A &= [u_1 \quad u_2 \quad u_3] \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ v_3^T \\ v_4^T \end{bmatrix} \\ &= \sum_{i=1}^{3} \sigma_i u_i v_i^T \end{aligned} \tag{45}$$

With Eq. (38), it can be shown that the pseudoinverse $A^+ = A^T(A A^T)^{-1}$ can also be expanded in terms of the singular vectors u_i and v_i as follows:

$$A^+ = V \Sigma^+ U^T \tag{46}$$

where

$$\Sigma^+ = \begin{bmatrix} 1/\sigma_1 & 0 & 0 \\ 0 & 1/\sigma_2 & 0 \\ 0 & 0 & 1/\sigma_3 \\ 0 & 0 & 0 \end{bmatrix} \tag{47}$$

Consequently, A^+ can be expanded in terms of the singular vectors u_i and v_i as follows:

$$A^+ = \sum_{i=1}^{3} \left(\frac{1}{\sigma_i}\right) v_i u_i^T \tag{48}$$

and we have

$$\dot{\delta} = A^+ u = \sum_{i=1}^{3} \left(\frac{1}{\sigma_i}\right) v_i u_i^T u \tag{49}$$

If the commanded torque vector u lies along one of the column vectors of U, that is, u_i, the gimbal rates computed by the pseudoinverse become

$$\dot{\delta} = A^+ u = (1/\sigma_i) v_i \tag{50}$$

where v_i is the ith column of V. Furthermore, if σ_i is zero, then the gimbal rate command for the commanded torque vector along u_i becomes infinity.

If rank(A) < 3 with nonzero singular values σ_1 and σ_2, we may directly use the Moore–Penrose generalized pseudoinverse defined as

$$A^+ = [v_1 \quad v_2 \quad v_3 \quad v_4] \begin{bmatrix} 1/\sigma_1 & 0 & 0 \\ 0 & 1/\sigma_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1^T \\ u_2^T \\ u_3^T \end{bmatrix}$$

$$= \sum_{i=1}^{2} \left(\frac{1}{\sigma_i}\right) v_i u_i^T \tag{51}$$

Note that the Moore–Penrose generalized pseudoinverse is defined for any rank-deficient matrix A, that is, when neither $(AA^T)^{-1}$ nor $(A^T A)^{-1}$ exists. The MATLAB® command for computing the Moore–Penrose generalized pseudoinverse is pinv(A).

The SR inverse $A^\# = A^T[AA^T + \lambda I]^{-1}$ can also be expanded as

$$A^\# = V\Sigma^\# U^T \tag{52}$$

where

$$\Sigma^\# = \begin{bmatrix} \sigma_1/(\sigma_1^2+\lambda) & 0 & 0 \\ 0 & \sigma_2/(\sigma_2^2+\lambda) & 0 \\ 0 & 0 & \sigma_3/(\sigma_3^2+\lambda) \\ 0 & 0 & 0 \end{bmatrix} \tag{53}$$

Consequently, $A^\#$ can be expanded in terms of the singular vectors u_i and v_i as follows:

$$A^\# = \sum_{i=1}^{3} \left(\frac{\sigma_i}{\sigma_i^2+\lambda}\right) v_i u_i^T \tag{54}$$

and we have

$$\dot{\delta} = A^\# u = \sum_{i=1}^{3} \left(\frac{\sigma_i}{\sigma_i^2+\lambda}\right) v_i u_i^T u \tag{55}$$

If the commanded torque vector u lies along one of the column vectors of U, that is, u_i, the gimbal rates computed by the singularity robust inverse become

$$\dot{\delta} = A^\# u = \sigma_i/(\sigma_i^2+\lambda) v_i \tag{56}$$

where v_i is the ith column of V. Furthermore, if σ_i is zero, then the gimbal rate command for the commanded torque vector along u_i becomes zero. At singular points, the gimbal rate command for the commanded u along the singular direction is zero. Consequently, singular points cannot be escaped simply using the standard singularity robust inverse.

Now consider the generalized singularity robust steering logic of the form

$$\dot{\delta} = A^\# u = A^T[AA^T + \lambda E]^{-1} u \tag{57}$$

where

$$E = \begin{bmatrix} 1 & \epsilon_3 & \epsilon_2 \\ \epsilon_3 & 1 & \epsilon_1 \\ \epsilon_2 & \epsilon_1 & 1 \end{bmatrix} > 0 \tag{58}$$

Because λ and ϵ_i are to be properly selected such that $A^\# u \neq 0$ for any nonzero constant u, the gimbal rate command will never become zero for any nonzero constant u, and, consequently, any internal singularities can be escaped using the proposed approach.

Numerical Example

Consider the typical pyramid mounting arrangement of four SGCMGs as was shown in Fig. 1. A skew angle β of 53.13 deg (i.e., $\cos\beta = 0.6$ and $\sin\beta = 0.8$) and constant unit momentum magnitude for each CMG are assumed. Initial gimbal angles are $\delta = (90, 0, -90, 0)$ deg, and the commanded torque vector is, given by $u = (-1, 0, 0)$. That is, the system is at an internal elliptic singularity and the commanded torque is along the singular vector direction.

For this case, the Jacobian matrix is simply given by

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & -0.6 & -1 & 0.6 \\ 0 & 0.8 & 0 & 0.8 \end{bmatrix} \tag{59}$$

Because rank(A) = 2 < 3, the standard pseudoinverse $A^+ = A^T(AA^T)^{-1}$ does not exist.

However, the singular value decomposition of A can be obtained using MATLAB as

$$A = U\Sigma V \tag{60}$$

where

$$U = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \tag{61a}$$

$$\Sigma = \begin{bmatrix} 1.6492 & 0 & 0 & 0 \\ 0 & 1.1314 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{61b}$$

$$V = \begin{bmatrix} 0.6063 & 0 & -0.7952 & 0 \\ 0.3638 & -0.7071 & 0.2774 & -0.5392 \\ 0.6063 & 0 & 0.4623 & 0.6470 \\ -0.3638 & -0.7071 & -0.2774 & 0.5392 \end{bmatrix} \tag{61c}$$

Furthermore, the Moore–Penrose generalized pseudoinverse can be obtained as

$$\text{pinv}(A) = \begin{bmatrix} 0 & -0.3676 & 0 \\ 0 & -0.2206 & 0.6250 \\ 0 & -0.3676 & 0 \\ 0 & 0.2206 & 0.6250 \end{bmatrix} \tag{62}$$

$$\Rightarrow \dot{\delta} = \text{pinv}(A) \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{63}$$

Because the commanded gimbal rates are zero, this singularity cannot be escaped via the Moore–Penrose generalized pseudoinverse.

The SR inverse with $P = I_3$ and $Q = 0.01 I_4$ can be obtained as

$$A^\# = [A^T A + 0.01 I_4]^{-1} A^T = A^T[AA^T + 0.01 I_3]^{-1}$$

$$= \begin{bmatrix} 0 & -0.3663 & 0 \\ 0 & -0.2198 & 0.6202 \\ 0 & -0.3663 & 0 \\ 0 & 0.2198 & 0.6202 \end{bmatrix} \tag{64}$$

$$\Rightarrow \dot{\delta} = A^\# \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{65}$$

Again, the commanded gimbal rates are zero. It can be further verified that this singularity cannot be escaped via the SR inverse with any diagonal weighting matrices P and Q.

Now consider the generalized SR inverse with $\lambda = 0.01$ and $\epsilon_i = 0.01$ as follows:

$$A^{\#} = A^T[AA^T + 0.01E]^{-1} \tag{66a}$$

$$A^{\#} = \begin{bmatrix} 3.6627e{-3} & -3.6630e{-1} & 2.8111e{-5} \\ -4.0037e{-3} & -2.1980e{-1} & 6.2017e{-1} \\ 3.6627e{-3} & -3.6630e{-1} & 2.8111e{-5} \\ -8.3990e{-3} & 2.1976e{-1} & 6.2014e{-1} \end{bmatrix} \tag{66b}$$

Consequently, the commanded gimbal rates become

$$\dot{\delta} = A^{\#}\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3.6627e{-3} \\ 4.0037e{-3} \\ -3.6627e{-3} \\ 8.3990e{-3} \end{bmatrix} \tag{67}$$

which is a nonzero vector, and the actual torque vector output becomes

$$\begin{aligned} u &= AA^{\#}\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ -9.9626e{-3} & 9.9634e{-1} & -7.6463e{-5} \\ -9.9221e{-3} & -3.5983e{-5} & -9.9225e{-1} \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 9.9626e{-3} \\ 9.9221e{-3} \end{bmatrix} \end{aligned} \tag{68}$$

The system is in fact escaping from the singularity by torquing the spacecraft along the pitch/yaw axes; however, the pitch/yaw disturbance torque is relatively small compared to the commanded torque along the roll axis. The cross-axis disturbance torque can be further reduced by selecting smaller off-diagonal elements ϵ_i of E. In general, a tradeoff will be required between robust singularity transit/escape and the resulting transient pointing errors. Note that it is not physically possible to escape from such an elliptic internal singularity by pure null motion without torquing the vehicle. For a given torque command of $(1, 0, 0)$ with constant λ and $\epsilon_i = \epsilon$, the actual torque output becomes approximately $(1 - \lambda, \epsilon\lambda, \epsilon\lambda)$ when $m \gg m_0$, and it becomes approximately $(0, \epsilon, \epsilon)$ when m becomes near m_0.

In the next section, we will present simulation results of a CMG-based attitude control system employing the proposed generalized SR inverse logic.

Simulation Results

Consider an agile spacecraft with a typical pyramid mounting arrangement of four SGCMGs as was shown in Fig. 1. A skew angle is selected as $\beta = 53.13$ deg (i.e., $\cos\beta = 0.6$ and $\sin\beta = 0.8$) and the constant momentum magnitude for each CMG is assumed as 1000 N · m · s. The gimbal-rate command limit of each CMG is assumed as 2 rad/s. It is also assumed that the attitude control bandwidth needs to be lower than 5 rad/s and the maximum slew rate less than 10 deg/s.

A spacecraft with the following nominal inertia matrix is considered:

$$J = \text{diag}(21400, 20100, 5000)\ \text{kg}\cdot\text{m}^2 \tag{69}$$

The transverse axes of this near-axisymmetric spacecraft are the roll and pitch axes. The symmetry axis with the smallest moment of inertia is the yaw axis, which is pointing toward a target. The commanded quaternion vector for a rest-to-rest, 47-deg roll-axis reorientation maneuver is given as $(q_{1c}, q_{2c}, q_{3c}) = (0.4, 0, 0)$. The time-optimal reorientation for this particular maneuver should ideally be completed in 8 s.

Two different sets of initial gimbal angles are considered: case 1, $\delta = (0, 0, 0, 0)$ deg, and case 2, $\delta = (90, 0, -90, 0)$ deg, an elliptic singular point. A better or preferred set of initial gimbal angles may be selected for this particular roll-axis maneuver. However, we consider here such worst cases of initial gimbal angles to demonstrate the effectiveness of the proposed control logic for passing through, and even escaping from, an internal elliptic singular point.

The proposed CMG steering logic based on the generalized SR inverse is integrated with a nonlinear attitude control logic developed in Ref. 14 as follows:

$$\tau = -J\left\{2k \,\underset{L_i}{\text{sat}}\left(e + \frac{1}{T}\int e\right) + c\omega\right\} \tag{70a}$$

$$L_i = \frac{c}{2k}\min\left\{\sqrt{4a_i|e_i|},\ |\omega_i|_{\max}\right\} \tag{70b}$$

$$A^{\#} = A^T[AA^T + \lambda E]^{-1} \tag{70c}$$

$$u = -\tau - \omega \times h \tag{70d}$$

$$\dot{\delta}_c = \underset{\circ\,\dot{\delta}_{\max}}{\text{sat}}\ \{A^{\#}u\} \tag{70e}$$

$$\dot{\delta} = \frac{(50)^2}{s^2 + 2(0.7)(50)s + (50)^2}\dot{\delta}_c \tag{70f}$$

where $\dot{\delta}_{\max} = 2$ rad/s, $|\omega_i|_{\max} = 10$ deg/s, and e is the quaternion-error vector defined in Eq. (7). A second-order gimbal dynamics is also included here.

When $\omega_n = 3$ rad/s, $\zeta = 0.9$, and $T = 10$ s, the controller gains k and c are chosen as $k = \omega_n^2 + 2\zeta\omega_n/T = 9.54$ and $c = 2\zeta\omega_n + 1/T = 5.5$. The control acceleration a_i is chosen as 40% of the actual maximum acceleration to accommodate the actuator dynamics and the nonlinear nature of quaternion kinematics.[14]

For the normalized Jacobian matrix A, the scale factor λ and E are chosen as

$$\lambda = 0.01\exp[-10\det(AA^T)] \tag{71a}$$

$$E = \begin{bmatrix} 1 & \epsilon_3 & \epsilon_2 \\ \epsilon_3 & 1 & \epsilon_1 \\ \epsilon_2 & \epsilon_1 & 1 \end{bmatrix} \tag{71b}$$

where $\epsilon_i = 0.01\sin(0.5\pi t + \phi_i)$ with $\phi_1 = 0$, $\phi_2 = \pi/2$, and $\phi_3 = \pi$.

As can be seen in Fig. 2 for case 1, the roll-axis reorientation maneuver of $(q_{1c}, q_{2c}, q_{3c}) = (0.4, 0, 0)$ is successfully completed within 8 s in the presence of the CMG singularity encounters, momentum saturation, and gimbal-rate limits. The effect of singularity-escaping torque disturbances on q_2 and q_3 during the internal singularity transit is evident in Fig. 2. However, the cross-axis,

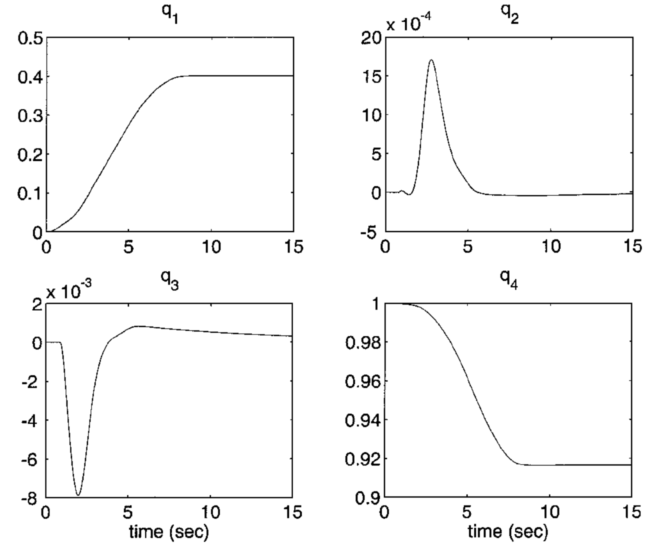

Fig. 2 Time histories of spacecraft attitude quaternions (case 1).

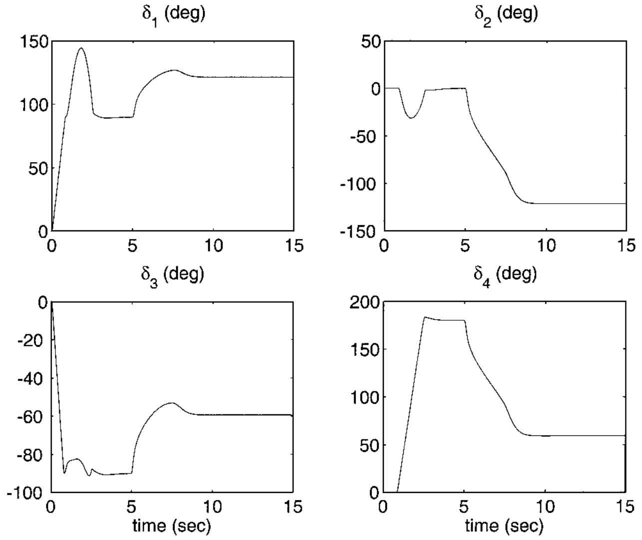

Fig. 3 Time histories of CMG gimbal angles (case 1).

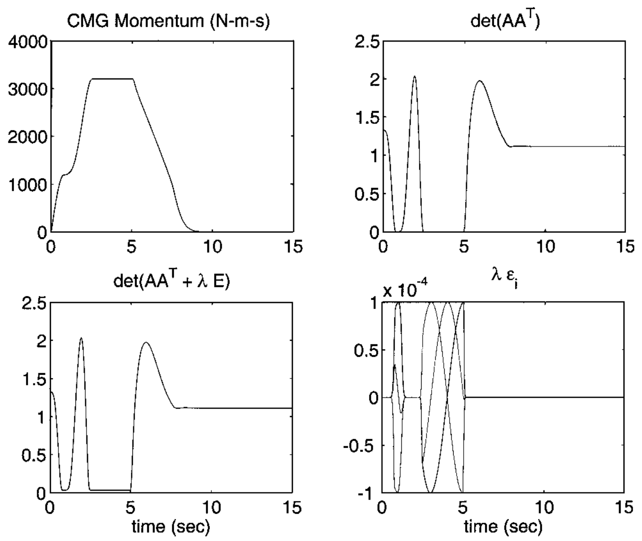

Fig. 4 Time histories of CMG momentum and singularity measures (case 1).

pitch/yaw pointing error during the singularity transit is relatively small compared to the roll maneuvering angle of 47 deg.

As can be seen in Fig. 3, the proposed CMG steering logic does not explicitly avoid singularity encounters; rather, it approaches and rapidly transits the internal elliptic singularity of $\boldsymbol{\delta} = (90, 0, -90, 0)$ deg. The plots of the total CMG momentum and the singularity measures, shown in Fig. 4, further indicate that the CMG system with the proposed control logic successfully passed through the internal elliptic singularity of $\boldsymbol{\delta} = (90, 0, -90, 0)$ deg to achieve near-minimum-time reorientation of the spacecraft. Because the proposed steering logic is, in fact, based on the minimum two-norm, pseudoinverse solution, it does not explicitly avoid singularity encounters; rather, it approaches and rapidly transits unavoidable singularities whenever needed, as demonstrated here.

For case 1, but with the standard singularity robust inverse with $\boldsymbol{E} = \boldsymbol{I}$, the system becomes trapped at the internal singularity of $\boldsymbol{\delta} = (90, 0, -90, 0)$ deg with the total CMG momentum of $1200\ \mathrm{N \cdot m \cdot s}$, even in the presence of sensor noises.

Case 2 was also simulated, but the simulation results are not presented here. Although the maneuver time was longer than that of case 1, the singularity escape capability of the proposed control logic, while minimizing its effect on the spacecraft attitude control, was verified for a rest-to-rest maneuver even starting at an internal elliptic singular point of $\boldsymbol{\delta} = (90, 0, -90, 0)$ deg.

As demonstrated by numerical simulations, the proposed control logic fully utilizes the available CMG momentum space in the presence of any singularities. However, the proposed control logic is mainly intended for typical reorientation maneuvers in which precision pointing or tracking is not required during a maneuver. Although there are special cases in which prescribed attitude trajectories are to be exactly tracked in the presence of internal singularities, most practical cases will require a tradeoff between robust singularity transit/escape and the resulting transient pointing error.

Conclusions

In this paper a simple yet effective way of transiting, and also escaping from, the internal elliptic singularities has been described. The generalized SR inverse was introduced, which utilizes a nondiagonal weighting matrix in the mixed, two-norm and weighted least-squares minimization problem. Such a singularity robust steering logic has been further integrated with a quaternion-feedback attitude control system for the SR, large-angle, near-minimum-time maneuvering control of an agile spacecraft. The simplicity and effectiveness of the proposed control logic has been demonstrated by numerical simulations.

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